IEA Third International Math and Science Study / Learning Environment Study

Video Study (Learning Environment Study)

8th Grade, Group 6 Math Lesson Plan

1. Subject material: Inequalities

2. On the subject material

(1) Students learn to express the relative size of quantities in the second grade of elementary school by using the inequality symbols " >" and "<". By the fifth grade, students are capable of comparing the quantity of fractions and comparing numerical values in general. In the sixth grade, we compile what has been studied about numerical values thus far, making sure students have a solid understanding of numerical equivalents and the less than / greater than relationship of quantities, while also furthering their understanding of the number line.

In the seventh grade, we go from examining the relationship of quantities to the use of algebraic equations, making sure the students understand the concept of the variable symbols and the solution. We reintroduce the characteristics of a simple linear equation and this is applied to learning how to solve algebraic equations. In connection to expressing the domain function, we introduce the use of the inequality symbols "≤" and "≥".

In the eighth grade, the concepts introduced within the study of algebraic equations are then applied as their studies develop to inequalities. By having the students see both algebraic equations and inequalities as a kind of mathematical statement that represents the conditions of an unknown number or the variable x, they can see algebraic equations and inequalities as integrated. In this way, the students' understanding of algebraic equations and inequalities are taken to an even deeper level. The solution to an inequality usually has countless solutions, except in the case when a variable has special limits. The solution therefore can be expressed as a set of numbers in a fixed range and I have them understand that this is a point where an inequality differs from a simple linear algebraic equation.

I introduce the solving methods of inequalities by again comparing and applying the characteristics of solving a simple linear equation.

(2) The instruction of inequalities is an important subject material from the point of view of thoroughly applying previously learned materials and taking the comprehension to an even deeper level. However, when considering the actual situation of the students, even if they are capable of basic calculations, the students often have difficulty grasping word problems and setting up mathematical statements for the relationship that is being expressed. In particular, they have a hard time with incorporating the variable symbols into the mathematical statement. Also, by the eighth grade, differences in the students'

abilities become more pronounced and I feel that there are quite a few students who do not fully understand what a mathematical statement is and how to set one up.

Because these conditions are the basis from which I have to teach inequalities, I cannot just simply teach it as a procedure. The students need to be taught that the relationship of the quantity in an actual problem is not always in an equal relationship, and that in many cases they have a less than / greater than relationship. It is important to engage the students enthusiastically during this process. I also try to devise creative ways to make it easier to comprehend this less than / greater than relationship by illustrating the problem and using other teaching materials and aids. I plan on allotting students plenty of time to think about the problem and figure it out on their own.

3. Connection to the research subject at this school

The research subject at this school was decided as " Improvement of Teaching Methods" and a great emphasis is being placed on efforts to promote individualized instruction as a measure to actualize this. As a mathematics instructor, I try to participate by paying attention to the following points in my everyday classes:

- Creative attention towards the problem: Creating problems that arouse the students' interest and motivation, and stimulates their mathematical thinking process as well. Problems that allow students to find a solving method according to his / her abilities.

- Steps towards problem solving: Ask questions and give supportive comments and advice to stimulate the students' thinking process. Use teaching materials and aids that support the thinking process. Be sure to allot "thinking-time".

4. Goals

Make the students able to express the relative size of quantities in inequalities and use inequalities to solve the problems as a procedure.

(1) Make the students able to express the less than / greater than relationship of quantities as an inequality. Make them understand the concept of the inequality and it's solution.
(2) Make the students understand the characteristics of the inequality and be able to use them to solve simple linear inequalities. Make them able to apply the use of inequalities to solve word problems.

5. Instruction Timeline

(1) Inequalities and their characteristics	3 hours (This class 1/3)
(2) How to solve inequalities	3 hours
(3) Problem	1 hour

6. Lesson for this class period

(1) Goals for this class period

I find occurrences from everyday life that contain a less than / greater than relationship and have the students try to express them using the inequality symbols. By comparing inequalities with algebraic equations, I hope to foster their thinking ability and an inquisitive attitude.

Students should:

- understand the usefulness of the inequality symbols and use them positively and with confidence. (interest, enthusiasm, attitude)
- be able to find examples of less than/greater than relationship in everyday life occurrences. (mathematical way of seeing and thinking)
- be able to use the inequality symbols to express the relative size of quantities (expression, solution)
- be able to understand the concept of the inequality and it's solution. (Knowledge, comprehension)

(2) Efforts to promote individualized instruction

1. Creative attention towards the problems

In this lesson, I decided on two problems which were concrete examples and would draw out the students' interests. Furthermore, I made the two problems be related to each other.

Problem 1, especially, is an example of how the students could solve the problems in a variety of methods according to their abilities, as they consider the less than / greater than relationship with the amounts of money. (Manual actions, use of tables, use of arithmetic, algebraic equations, inequalities, etc.) I want the students to be able to consider a wide variety of solution methods, appreciating the value of each method and coming to deeper understanding of algebraic equations and inequalities. Also, I want the students to notice that the inequality in the first problem has a limited range of solutions which is one of the ways inequalities are characteristically different from an algebraic equation.

With Problem 2, I thought up a problem that deals with the relative size of quantities and is easy to set up as a mathematical statement. I plan to use a number line so the students can visually comprehend that there are an infinite number of solutions to this problem. I believe that by examining these two problems, the students will have a deeper understanding of what the solution of an inequality entails.

2. Steps towards problem solving

A. Stage of understanding the problem

In Problem 1, I have the students think about what the less than / greater than relationship is for the amount of money in the wallets if the exact same number of coins are taken out of both the older brother's wallet (he only has 10 yen coins) and the younger brother's wallet (he only has 5 yen coins). I believe this is a kind of problem that hits close to home for the students.

First of all, I ask the students what the remaining sum is as I remove each coin from the teaching aid. I do the same as I remove two coins at a time. After this point, I have the students work on Problem 1 on their own because I feel that the above method of removing the coins one at a time, then two at a time, becomes a sufficient enough clue to finding the solution as a matter of chronological progression (manual actions, using a table, etc.).

B. Stage of pursuing the problem

During the problem solving stage, I ask the students who have found the solution by thinking of it in terms of chronological progression if there could be an even faster method to finding the solution. I ask the students who found the answer through arithmetic if they could set up an algebraic equation by thinking in terms of chronology. I ask the students who solved the problem using an algebraic equation to consider even further if there is a more suitable way to represent the problem as mathematical statement and have them think about problem from many different angles. As I make my instructional class rounds, I look for good examples of the various methods- manual actions, use of tables, use of arithmetic, use of algebraic equations- and have them presented to the class. If there is a student who has solved the problem using inequalities, the focus is placed on how the mathematical statement is set up and getting too deeply into the how to actually solve the problem is avoided for the time being.

While the students are learning the various solving methods, I have the students understand that it is not actually correct to represent the problem as an algebraic equation and that the use of the inequality within the mathematical statement is the most appropriate for the problem.

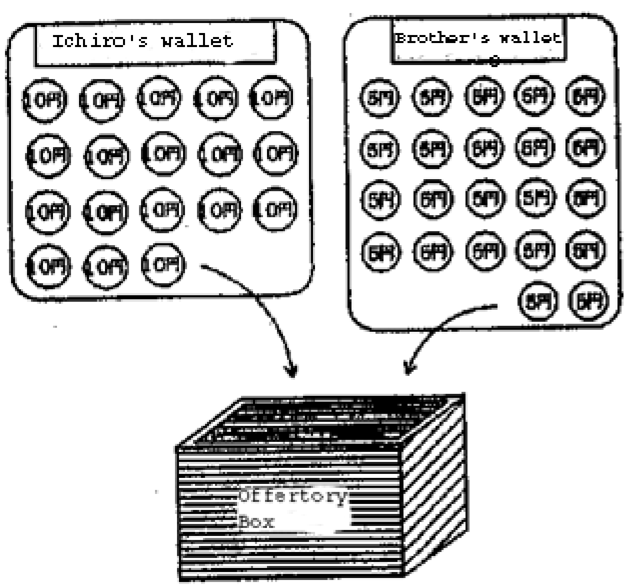
C. Stage of solving the problem, reflecting, and class presentations

I explain the solving method of plugging in a range of numbers in order to define what the solution for an inequality entails. Then, I bring out a table to help the students think about the solution more concretely. As we practice the calculations, we compare the values on each side and write in the proper inequality symbol. As a result, I have the students notice that this inequality has 4 possible answers and therefore is different in nature from an algebraic equation (which has only one possible answer). In Problem 2, with the inequality dealing with amounts, I use a number line to have the students understand that there can be an infinite number of solutions for this problem.

3. Evaluation

- For Problem 1, did you try to the best of your ability to find a solution method and try to tackle the problem? Did you try to consider if there was an even better solution method?
- Are you able to express a simple less than/greater than relationship of quantity in an inequality and figure out its solution?

4) This period's structure

<table>
<tr><th>LESSON FLOW</th><th>EXPECTED STUDENT REACTIONS</th><th>TEACHER'S ACTIONS</th><th>WAYS OF THINKING</th></tr>
<tr><td>1.Present the problem</td><td></td><td>The problem is distributed as a hand-out</td><td></td></tr>
<tr><td colspan="4">Problem 1:
It has been one month since Ichiro's Mother has entered the hospital. He has decided to pray with his younger brother at a local temple every morning so that she will get better soon. There are 18 ten-yen coins in Ichiro's wallet and just 22 five-yen coins in the younger brother's wallet. They have decided to take one coin from each wallet everyday and put them in the offertory box and continue to pray until either wallet becomes empty. One day, when they looked in to each other's wallets when they were done with their prayer, the younger brother's amount was greater than in Ichiro's. When this happened, how many days had it been since they started their prayers?

</td></tr>
<tr><td>2. Comprehension of the problem.

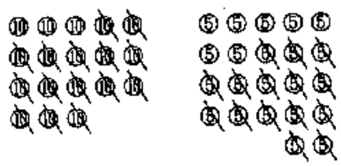
3.Steps toward solving the problem</td><td></td><td>I present an illustration to make the meaning of the problem easier to understand.</td><td>1. Understand that you should figure out the two brothers' remaining balances and compare them.</td></tr>
</table>

<table>
<tr>
<td>2. Who has more money on the first day?

• How about the second day?

• On what day did the younger brother realize he had more money?
(Each student thinks about the above independently)</td>
<td>• On the first day, Ichiro has more money.

• Ichiro has more money on the second day as well.

A.

The students track the remaining balance on their handout illustrations by putting a mark on the same number of coins for each of the brothers. They will find that the younger brother has more money on the <u>15th day</u>.</td>
<td>

• I give the students plenty of time to work out the problem on their own. I observe the students' reactions closely.

• I have the students who solved the problem through chronological thinking (A,B), consider if there could be a faster method.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B.
<table>
<tr><td>Number of Days</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td></tr>
<tr><td>Ichiro's Balance</td><td>170</td><td>160</td><td>150</td><td>140</td><td>130</td><td>120</td><td>110</td><td>100</td><td>90</td><td>80</td><td>70</td><td>60</td><td>50</td><td>40</td><td>30</td><td>20</td><td>10</td><td>0</td></tr>
<tr><td>Brother's Balance</td><td>105</td><td>100</td><td>95</td><td>90</td><td>85</td><td>80</td><td>75</td><td>70</td><td>65</td><td>60</td><td>55</td><td>50</td><td>45</td><td>40</td><td>35</td><td>30</td><td>25</td><td>20</td></tr>
</table>
<u>From the 15th day forward.</u></td>
<td>• There are probably many students who think the answer is <u>only on the 15th day</u>. I do not go too deeply into it at this stage and have them think about it when we are examining inequalities.</td>
<td>3. You can figure this out by chronological thinking.
A. Manual Actions
B. Using a table</td>
</tr>
<tr>
<td></td>
<td>C. 180-110=70
10-5=5
70 ÷ 5=14
14+1=15

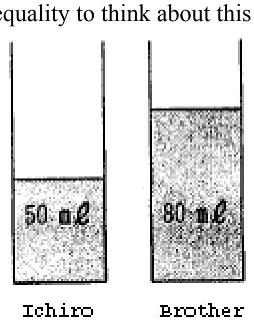
<u>From the 15th day forward.</u></td>
<td>• I have the students who solved the problem through arithmetic try applying chronological thinking to setting up an equation.</td>
<td>4. Pay attention to the fact that the difference between the two brothers' remaining balances decreases by 5 yen each day.
C. Using arithmetic</td>
</tr>
<tr>
<td></td>
<td>D. 180-10x=110-5x
-10x+5x=110-180
-5x=-70
x=14
14+1=15

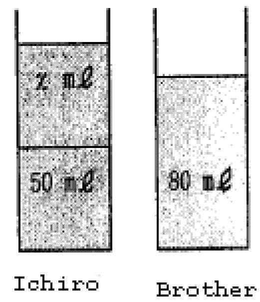
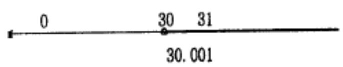
<u>From the 15th day forward</u>.</td>
<td>1. I have the students who solved the problem using an algebraic equation consider if there is a more suitable mathematical statement that can represent the problem.</td>
<td>5. Apply previously learned concepts.
D. Use of equation</td>
</tr>
</table>

<table>
<tr>
<td>4. Present how to solve the problem.</td>
<td></td>
<td>I choose students who are using the 4 representative solving methods to present them to the class.
In the case there is a student who used an inequality to solve the problem, I have them present this to the class with an emphasis on just setting up the inequality (and not on solving it).</td>
<td></td>
</tr>
<tr>
<td>5. Think about how to express the mathematical statement within the problem.

6. Learn the definition of an inequality.

7. Learn the definition of a solution to an inequality by thinking about which numerical values satisfy the inequality and which do not.</td>
<td>E. 180-10x<110-5x
<table>
<tr><td></td><td>x</td><td>$180-10x$</td><td></td><td>$110-5x$</td></tr>
<tr><td>×</td><td>13</td><td>180-10×13=50</td><td>></td><td>110-5×13=45</td></tr>
<tr><td>×</td><td>14</td><td>180-10×14=40</td><td>=</td><td>110-5×14=40</td></tr>
<tr><td>○</td><td>15</td><td>180-10×15=30</td><td><</td><td>110-5×15=35</td></tr>
<tr><td>○</td><td>16</td><td>180-10×16=20</td><td><</td><td>110-5×16=30</td></tr>
<tr><td>○</td><td>17</td><td>180-10×17=10</td><td><</td><td>110-5×17=25</td></tr>
<tr><td>○</td><td>18</td><td>180-10×18= 0</td><td><</td><td>110-5×18=20</td></tr>
</table>
The solution for the inequality is {15, 16, 17, 18}
so the answer to Problem 1 is: One of the following days: the 15th, 16th, 17th or 18th day.</td>
<td>While calculating the plugged in numbers, I compile the less than / greater data for both sides into a table.
I have the students notice that the solution to E has a range of answers unlike an algebraic equation (which only has one answer).</td>
<td>Notice that an algebraic equation is inadequate as a numerical expression to represent the problem.
6. Convert the word problem into a mathematical statement and think about it.

7. Notice that there are more than one solution to the inequality.</td>
</tr>
<tr>
<td>8. Present Problem 2</td>
<td></td>
<td>Distribute the handout for Problem 2.</td>
<td></td>
</tr>
<tr>
<td colspan="4">Problem 2:
The prayer was answered and their mother was able to leave the hospital in health. That night they made a toast with juice. Presently, there are 50 ml of juice in Ichiro's cup, and 80 ml of juice remaining in the younger brother's cup. When the mother poured more juice into Ichiro's cup, he now had more juice than his brother. How many ml did the mother pour? Let's set up an inequality to think about this problem.

</td>
</tr>
<tr>
<td>9. Comprehension of the problem and planning the solution</td>
<td></td>
<td>I explain the meaning of the problem using a teaching</td>
<td></td>
</tr>
</table>

		aid.	
(1) What to represent as x in setting up the inequality.	• The amount of juice which was added • 50+x>80		• Replace the variable with a letter.
• Set up a mathematical statement.	 	• I have the students pay close attention to the numerical values and the variable letters on the illustrations and have them set up a mathematical statement.	• Visually assess the various amounts and then represent less than/greater than relationship into a mathematical statement.
• Find the solution	• 80-50=30 <u>a number greater than 30</u> • 50+x=80 x=30 <u>a number greater than 30</u>		8. Checking the solution. • Use of an algebraic equation.
• Closely examine the solution	 50+31>80 50+30.001>80 • If the number is even slightly greater than 30, it is a possible solution. Therefore, the answer to Problem 2 is: <u>an amount greater than 30 ml.</u>	• I use a number line to help the students understand that even if the amount is only slightly greater than 30, it satisfies the inequality.	• Notice how when you have expressed the relative size of quantities as an inequality, the possibility of solutions for this problem are infinite.
10. Summarize this period's lesson.		• Through Problems 1 and 2, I have the students understand that depending on the value of x, the inequality can have a limited range of solutions or an unlimited possibility of solutions.	

LESSON FLOW (question, etc..)

♦…Student responses

- Distribute the handout for Problem 1 and read it.
- Present the paper model (illustrated teaching aid) to the class.

"OK. Let's go over the problem. There are only 10 yen coins in Ichiro's wallet. How much did he have all together?

♦180 yen

"How much was in the younger brother's wallet?"

♦ 110 yen

"In the beginning stage, Ichiro had more money in his wallet, right? After this point, they put in one coin each into the offertory box everyday. Now, let's say they have just offered their first prayer on the first day. At this stage, which wallet has more money in it?"

♦ The older brother's

"How about after the second's day's prayer is finished"

♦ The older brother still has more"

"One day, when they looked into each others' wallets, the younger brother had more. How many days had past since they began their prayers? Think about this on your own and try to figure it out."

....

(Instructional classroom rounds)

- To the student who cannot figure out any solution methods:
  "How about on the 3rd day? How about the 4th day?"
- To the students who are manipulating actual objects and using a table:
  "Are there any faster methods to solving the problem?"
- To the students who are using arithmetic:
  "Can you think of a way the contents of the wallets could be directly expressed in a mathematical equation?"
- To the students who used an algebraic equation to solve the problem:
  "Can you set up a mathematical statement that is even more fitting for the problem?
- To the students who used inequalities to solve the problem:
  "Can you find the solution using a method we have learned previously?"

....

- Choose students who used the various methods (manual actions, table, arithmetic, algebraic equations) and have them put their work up on the chalkboard and present their ideas to the class.

- I attach the following signs next to the corresponding examples as they are presented :

While manipulating actual objects

Using a graph

Because the difference shrinks by 5 yen at a time

If after x days the monetary amount becomes the same…

.

“Please raise your hands if you used this method, or if you would like to use this method. How many of you are there?”

∗ In the case of the table, I have them make an abridged version.

1	2	3		13	14	15	16	17	18
170	160	150	....	50	40	30	20	10	0
105	100	95	....	45	40	35	30	25	20

- I call on a student who has used an inequality, and have the student immediately present just the mathematical statement only (the inequality itself without its solution.)

I attach a sign on the board that reads:

If the younger brother has more money after x number of days

“How many people set up this kind of mathematical statement?”

In the case there are no students who used inequalities to solve the problem

(While looking at an algebraic equation) “Let’s think about whether the problem can be represented in a mathematical statement that is even more fitting. The contents of Ichiro’s wallet after x number of days is 180-10x (yen) and the contents of the younger brother’s wallet is 110-5x (yen), right? If the younger brother now has more money after x number of days, what kind of symbol would go here?”

“A mathematical statement which uses the inequality symbol in this way is called an inequality.”

- I have the students write out the inequality and the term on their handout. (During this time I write out the range on the chalkboard.)

“Now, let’s try to find the numerical values for x that satisfy the inequality. In the other solution methods, the answer was on the 15th day. So let’s plug in some numbers that are greater and less and work on the problem. Let’s compare the results for the numbers from about 13 to 18, as some examples of numbers close to 15.”

“If we plug in 13 for x on the left side of the inequality., then 180-10x13 = ? ”

♦50

“If we plug in 13 for x on the right side of the inequality, then 110-10x13= ? ”

♦45

“Since the left side of the inequality is greater, the greater than symbol goes here and this does not match the above inequality. So we can say 13 is not a numerical value that can satsify the above inequality.”

- Put an x on the left side of 13 (to indicate that it is not a possible solution).

“Next, please calculate when x is 14, 15, and so on in the same manner and compare the results.”

....

Choose someone to fill in the table on the chalkboard.

"Who got the same answers?"
"This means, the numerical value which can satisfy the above inequality is not only 15, but 16 and 17 as well. If you look at the less than / greater than column, the left and right side become equal at 14 and after that, the inequality symbol starts to face in the same direction as in the above inequality. So, the numerical value which satisfies the above inequality has to be a number which is what?"

♦A number greater than 14

"The numerical value for this x is called a solution to the above inequality. It is the numerical value that satisfies the mathematical expression. So this is the same as in the case with an algebraic equation."
"Now, about the answer to this problem. If you look at this table, 15, 16, 17 and 18 are all possibilities, but can an even greater number be a possible answer? For example, what about on the 19th day?"

♦The older brother's wallet will be empty by the 18th day so they cannot go on with their prayers. So the 19th day is not a fitting answer.

"So that means, it had to be one of the following days: 15th, 16th, 17th or 18th."

- Distribute the handout for Problem 2 and read it.
- Present the illustration (paper constructed teaching aid) to the class.

"Now, let's think about the problem by looking at the illustration..."
"We are trying to think in terms of inequalities, so what should we represent with x?"

♦The amount of juice that was added

- Have the students write in "x ml" on the illustration on their handouts.

"Please use an inequality to express how Ichiro now has more juice than his younger brother.

♦$50+X>80$

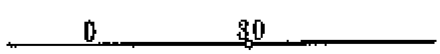
"What is the solution to this inequality? In other words, what numerical value for x would satisfy this statement?"

♦A number greater than 30. A number greater than 31.

If there is no reaction/response from the students
"In order to fulfill this expression, can you think of a number that could work for an example?"

In the case that the response is: "A number greater than 30"

- Draw a number line



"A number greater than 30. Can you give me an example?"

♦31

"How about another?"

♦32

"Can we say that a number smaller than 31, like 30.5 is a possible solution to the inequality?"

♦ Yes

"Now then, how many numerical values could satisfy this mathematical statement?"

♦ There are an infinite number of possibilities."

In the case that the response is: " A number greater than 31"

- Draw a number line

"So you are saying the smallest numerical value which could satisfy the statement is 31. Is this right?

♦ No. It can be any number that is even slightly greater than 30.

"For example?"

♦ 30.1, 30.02, ..."

"Now then, how many numerical values could satisfy this mathematical statement?"

♦ There are an infinite number of possibilities.

"Therefore, the answer to this problem is any amount greater than 30 ml."

"I will now review what we studied today. We learned that we can use inequalities when representing the two problems in a mathematical statement. Also, we learned that an inequality is not limited to one solution as is in the case of an algebraic equation. Specifically, there is the case when there is a limited range of solutions as in Problem 1 or there is the case where there are an unlimited number of possibilities as in Problem 2."